SEC 3 ADD MATH
CHAPTER 6.1: EXPONENTIAL AND LOGARITHMIC FUNCTIONS

A6 Exponential Functions
A6.1 Exponential functions including
• laws of Indices
• exponential equations with the same base

Learning Outcomes
By the end of this lesson, you should feel confident to
- Simplify expressions involving exponents
- Apply the laws of indices
- Solve exponential equations

(A) Laws of Indices

The symbol "e" commonly represents Euler's number, also known as Euler's constant or the base of the natural logarithm. It is an irrational number approximately equal to 2.71828.

The laws governing exponents are applicable to "*e*" as a distinct base, representing a special instance within the rules of indices.

Law 1	$a^m \times a^n = a^{m+n}$	$e^m \times e^n = e^{m+n}$
Law 2	$a^m \div a^n = a^{m-n}$ *or* $\frac{a^m}{a^n} = a^{m-n}$	$e^m \div e^n = e^{m-n}$ *or* $\frac{e^m}{e^n} = e^{m-n}$
Law 3	$(a^m)^n = a^{mn}$	$(e^m)^n = e^{mn}$
Law 4	$a^m \times b^m = (a \times b)^m$	$e^m \times b^m = (e \times b)^m$
Law 5	$a^m \div b^m = (a \div b)^m$	$e^m \div b^m = (e \div b)^m$
Zero Indices	$a^0 = \frac{a^1}{a^1} = \frac{a^n}{a^n} = 1$	$e^0 = \frac{e^1}{e^1} = \frac{e^n}{e^n} = 1$

Negative Indices	$a^{-n} = \frac{1}{a^n}$ $\left(\frac{a}{b}\right)^{-n} = \left(\left(\frac{b}{a}\right)^{-1}\right)^{-n}$ $= \left(\frac{b}{a}\right)^{-1\times -n}$ $= \left(\frac{b}{a}\right)^{n}$	$e^{-n} = \frac{1}{e^n}$ $\left(\frac{e}{b}\right)^{-n} = \left(\left(\frac{b}{e}\right)^{-1}\right)^{-n}$ $= \left(\frac{b}{e}\right)^{-1\times -n}$ $= \left(\frac{b}{e}\right)^{n}$
Exponential equation (with common base)	If $a^m = a^n$, where $a \neq -1, 0, 1$ Then $m = n$	If $e^m = e^n$ Then $m = n$

Example 1

Simplify the following expressions, leave your answer in positive index.

(a) $e^2 \cdot e^{-5}$

(b) $\frac{4e^{-6}}{16e^3}$

(c) $(e^x)^6 \times (e^2)^x \div (e^5)^x$

(d) $\left(\frac{5e^3}{3e^{-2}e^6}\right)^{-2}$

(e) $\left(\frac{-6e^6}{3e^5}\right)^3 \div \left(\frac{e^7}{e^3}\right)^5$

Example 2

Solving exponential equations

Solve each of the following exponential equations.

(a) $5^{2a} \times 4^{a} = \frac{1}{10}$

(b) $\left(\sqrt[3]{e}\right)^{2x} \div \left(\frac{e^2}{e^6}\right)^{-x} = 1$

Example 3

Solving exponential equations using substitution

Solve each of the following exponential equations by substitution.

(a) $5^{2x+1} + 9(5^x) = 2$

(b) $e^{2x} = 2e^2 - e^{x+1}$

Example 4

Solving simultaneous exponential equations

Solve the simultaneous equations, $\frac{4^y}{2^{5x}} = 256$ and $5^y = 125(25^x)$.

Answers

1(a) $\frac{1}{e^3}$	(b) $\frac{1}{4e^9}$	(c) e^{3x}	(d) $\frac{9}{25}e^2$	(e) $-\frac{8}{e^{17}}$
2(a) $-\frac{1}{2}$	(b) 0	3(a) -1	(b) 1	(4) -2, -1

(E) Practice Questions

1. Solve each of the following exponential equations.

   (a) $4^x + 4^{x+2} = 272$ (b) $3^{x+1} + 3^x = 12\sqrt{3}$

2. Solve the following equations.

   (a) $9^{x+1} + 6(3^x) = 3$ (b) $2^{2x-1} + 1 = 9(2^{x-2})$

SIRIUS MATHEMATICS

3. Solve the following pair of simultaneous equations.

(a) $2^x - 3^y = 119$ and $2^{x-3} + 3^{y-2} = 17$

(b) $5^{x-3} = \frac{5^y}{25}$ and $3^x + 3^y = \frac{4}{9}$

*4. Using substitution, or otherwise, solve $\frac{1}{2^{2x-1}} - \frac{1}{2^{x-1}} - \frac{1}{2^x} + 1 = 0$.

(Sample of Higher Order Thinking Question)

Answers

1(a) 2	(b) 1.5	2(a) -1	(b) 2 or -1
3(a) $x = 7, y = 2$		(b) $x = -1, y = -2$	(4) Review in class

(E) More Practice Questions (Self Practice Questions)

1. Solve each of the following exponential equations.

(a) $4(2^{x+2}) = 2^{x+2} + 384$

(b) $2^x + \sqrt{2^x} - 6 = 0$

2. Solve the following pair of simultaneous equations.

$4^x \times 2^y = 1024$ and $3^x \times 81^y = \frac{1}{9}$

Answers

1(a) 5 (b) 2 (2) $x = 6, y = -2$

(G) Review Question **(Quiz at the end of lesson to check understanding)**

1. Solve the exponential equation $10(25^x) - 7(5^{x+1}) = 75$.