SEC 2 MATH
CHAPTER 4.2: FACTORISATION USING ALGEBRAIC IDENTITIES

N5. Algebraic expressions and formulae
N5.12. use of Algebraic Identities

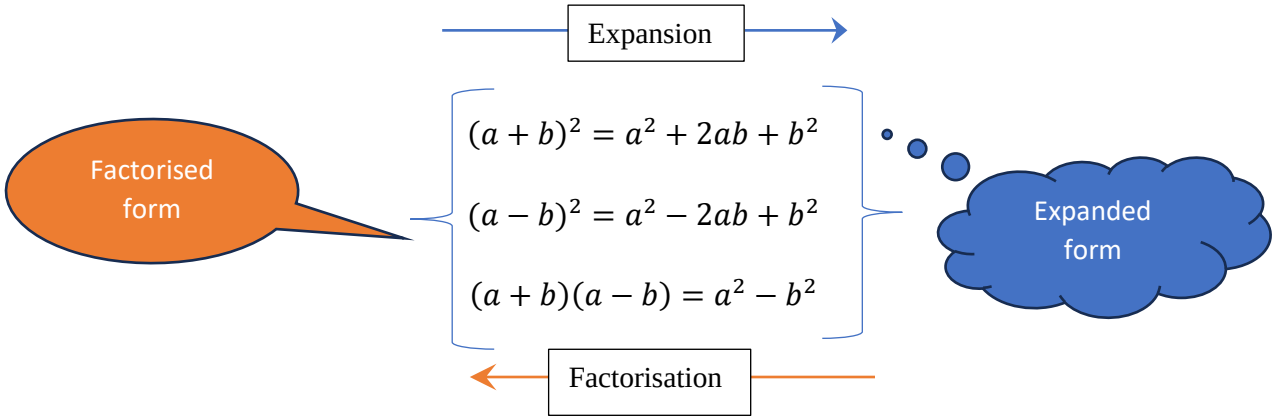
Learning Outcomes
By the end of this lesson, you should feel confident to
- factorise algebraic expression using the three special identities,
- factorise algebraic expression.

(A) Special Algebraic Identities

The three **Special Algebraic Identities** mentioned in the previous worksheet would be utilised.

- The square of sum: $(a+b)^2 = a^2 + 2ab + b^2$
- The square of difference: $(a-b)^2 = a^2 - 2ab + b^2$
- The difference of square: $(a+b)(a-b) = a^2 - b^2$

The Identities provided both the expanded and factorised form.



(B) Practice Questions

1. Factorise each of the following completely.

(a) $49a^2 + 42a + 9$ (b) $121y^2 + 220xy + 100x^2$

(c) $25h^2 + 64k^2 + 80hk$ (d) $25p^2 - 70pq + 49q^2$

(e) $121x^2y^2 - 22xy + 1$ (f) $121a^2 - 64b^2$

2. Factorise each of the following completely.

(a) $600x^2 - 120x + 6$ (b) $16x^2 - 100y^2z^2$

(c) $225h^2k^4 - 64g^2$ (d) $98ab^2 - 18a$

(e) $(3x + 5)^2 - 4x^2$ (f) $(6x - 1)^2 - (5x + 4)^2$

*(g) $x^4 - 1$

3. Without using a calculator, use algebraic rules to find the value of each of the following.

(a) $197^2 + 1182 + 9$ (b) $223^2 - 123^2$

(c) $0.53^2 - 1.47^2$ (d) $98^2 - 4$

4. It is given that $m^2 + \frac{1}{m^2} = 6$ and $m \neq 0$. Find the value of

(a) $m - \frac{1}{m}$,

(b) $\left(m + \frac{1}{m}\right)^2$.

(C) Summary of factorisation

Factorisation using algebraic identities involves breaking down **Quadratic** expressions into their simplest forms or factors using different techniques.
There are several key algebraic identities and methods for factorisation, including:

Factorise the following algebraic expressions.

1. Common factor: Identifying and factoring out the greatest common factor (GCF) from all terms of the expression.	$6x^2 + 12x$ =
2. Difference of squares: $a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9$ $= x^2 - 3^2$ =
3. Perfect square: $a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$x^2 + 6x + 9$ $= x^2 + 6x + 3^2$ $= x^2 + 2(x)(3) + 3^2$ = $x^2 - 10x + 25$ $= x^2 - 10x + 5^2$ $= x^2 - 2(x)(5) + 5^2$ =
4. Factoring by grouping: Arrange into groups that have a common factor.	$2x - 4 + ax - 2a$ $= 2(x - 2) + a(x - 2)$ =
5. Quadratic factorisation: For expressions of the form $ax^2 + bx + c$.	$x^2 + 5x + 6$ =

Flowchart for Algebraic Factorisation

First look for highest common factor (HCF). Base on the number of terms it could be one of the following factorisations.					
2 Terms	**3 Terms**		**4 Terms**		
Difference of squares	Perfect Squares	Quadratic factorisation (Use of multiplication frame)	Grouping 2 by 2	Grouping 3 by 1	Grouping 1 by 3

5. Factorise each of the following expressions completely.

(a) $(a+2b)^2-4b^2$

(b) $98a^2-84ab+18b^2$

(c) $6a^2b-5ab-6b$

(d) a^3+a^2-a-1

(e) $a^2-10a+25-25b^2$

(f) $a^2-b^2-2bc-c^2$

*6. Factorise completely $27a^2 - 3b^2$.

The solution by Student A is as follows:

$27a^2 - 3b^2$
Step 1: $= 3(9a^2 - b^2)$
Step 2: $= 3(9a - b)(9a + b)$

Identify the incorrect step. Provide a clear explanation using mathematical principles or rules.

Answers:

1(a) $(7a + 3)^2$ (b) $(11y + 10x)^2$ (c) $(5h + 8k)^2$
(d) $(5p - 7q)^2$ (e) $(11xy - 1)^2$ (f) $(11a - 8b)(11a + 8b)$
2(a) $6(10x - 1)^2$ (b) $4(2x - 5yz)(2x + 5yz)$ (c) $(15hk^2 - 8g)(15hk^2 + 8g)$
(d) $2a(7b - 3)(7b + 3)$ (e) $5(x + 5)(x + 1)$ (f) $(x - 5)(11x + 3)$
(g) $(x - 1)(x + 1)(x^2 + 1)$
3(a) 40000 (b) 34600 (c) -1.88 (d) 9600 4(a) $2\ or\ -2$ (b) 8
5(a) $a(a + 4b)$ (b) $2(7a - 3b)^2$ (c) $b(2a - 3)(3a + 2)$
(d) $(a - 1)(a + 1)^2$ (e) $(a - 5 - 5b)(a - 5 + 5b)$ (f) $(a - b - c)(a + b + c)$

(D) More Practice Questions

1. Factorise each of the following expressions completely.
   (a) $4x^2 - 16x^2y^2$ (b) $x^2 - 6xy + 9y^2$
   (c) $(3a + b)^2 - (2a - b)^2$ (d) $5a^2b^2 - 9ab - 2$
   (e) $12q^2 - 62q + 70$ (f) $18a^2 - 24ak - 12bk + 9ab$

2. Without using a calculator, use algebraic rules to find the value of each of the following.
   (a) $203^{\,2} - 1218 + 9$ (b) $\frac{25}{36} - (\frac{7}{42})^2$
   (c) $2.71^2 - 0.29^2$ (d) $2(98^2) - 8$

3. Given that $(u + v)^2 = 6$ and $uv = 1.5$, find the value of $\left(\frac{u-v}{3}\right)^2$.

4. Factorise $x^3 - x^2 - x + 1$.
   Hence factorise $\frac{8}{27}a^3 + \frac{4a^2}{9} - \frac{2}{3}a - 1$.

Answers:

1(a) $4x^2(1 - 2y)(1 + 2y)$ (b) $(x - 3y)^2$ (c) $5a(a + 2b)$
(d) $(5ab + 1)(ab - 2)$
(e) $2(3q - 5)(2q - 7)$ (f) $3(2a + b)(3a - 4k)$
2(a) 40000 (b) $\frac{2}{3}$ (c) 7.26 (d) 19200
(3) 0
(4) $(x - 1)^2(x + 1)$; $\left(-\frac{2a}{3} - 1\right)^2\left(\frac{2a}{3} - 1\right)$ or $\left(\frac{2a}{3} + 1\right)^2\left(\frac{2a}{3} - 1\right)$

(E) Review Question

1. Factorise each of the following expressions completely.
   (a) $2250 - 490x^2$
   (b) $169a^2 - 52a + 4$

(a) (b)

2. Factorise $21c^2 - cd - 2d^2$.
   *Hence factorise $84\left(\frac{7x}{2} + 4y\right)^2 - 42xz - 48yz - 72z^2$.

(D) More Practice Questions Answers

1(a) $4x^2 - 16x^2y^2$

$= 4x^2(1 - 4y^2)$

$= 4x^2(1 - 2y)(1 + 2y)$

(b) $x^2 - 6xy + 9y^2$

$= x^2 - (2)(x)(3y) + (3y)^2$

$= (x - 3y)^2$

(c) $(3a + b)^2 - (2a - b)^2$

$= (3a + b + 2a - b)(3a + b - 2a + b)$

$= 5a(a + 2b)$

(d) $5a^2b^2 - 9ab - 2$

$= (5ab + 1)(ab - 2)$

$5ab$	$+1$	$+ab$
ab	-2	$-10ab$
$5a^2b^2$	-2	$-9ab$

(e) $12q^2 - 62q + 70$

$= 2(6q^2 - 31q + 35)$

$= 2(3q - 5)(2q - 7)$

$3q$	-5	$-10q$
$2q$	-7	$-21q$
$6q^2$	$+35$	$-31q$

(f) $18a^2 - 24ak - 12bk + 9ab$

$= 18a^2 + 9ab - 24ak - 12bk$

$= 9a(2a + b) - 12k(2a + b)$

$= (2a + b)(9a - 12k)$

$= 3(2a + b)(3a - 4k)$

2. (a) $203^2 - 1218 + 9$

$= 203^2 - 2(203)(3) + 3^2$

$= (203 - 3)^2$

$= 200^2$

(b) $\frac{25}{36} - (\frac{7}{42})^2$

$= \left(\frac{5}{6}\right)^2 - (\frac{1}{6})^2$

$= (\frac{5}{6} + \frac{1}{6})(\frac{5}{6} - \frac{1}{6})$

$= (1)(\frac{4}{6})$

= 40000

(c) $2.71^2 - 0.29^2$

$= (2.71 + 0.29)(2.71 - 0.29)$

$= (3)(2.42)$

$= 7.26$

$= \frac{2}{3}$

(d) $2(98^2) - 8$

$= 2(98^2 - 4)$

$= 2(98^2 - 2^2)$

$= 2(98 + 2)(98 - 2)$

$= 2(100)(96)$

$= 19200$

3. $(u + v)^2 = 6$

$u^2 + 2uv + v^2 = 6$

$u^2 + v^2 + 2(1.5) = 6$

$u^2 + v^2 = 3$

$\left(\frac{u-v}{3}\right)^2 = \frac{u^2-2uv+v^2}{9}$

$= \frac{u^2+v^2-2uv}{9}$

$= \frac{3-2(1.5)}{9}$

$= 0$

4. $x^3 - x^2 - x + 1$

$= x^2(x - 1) - (x - 1)$

$= (x - 1)(x^2 - 1)$

$= (x - 1)(x + 1)(x - 1)$

$= (x + 1)(x - 1)^2$

$\frac{8}{27}a^3 - \frac{4a^2}{9} - \frac{2}{3}a + 1$

$= (\frac{2}{3}a)^3 - (\frac{2}{3}a)^2 - \frac{2}{3}a + 1$

Replace "x" with "$\frac{2}{3}a$".

$= (\frac{2}{3}a + 1)(\frac{2}{3}a - 1)^2$